Midwest Dynamical Systems 2021 (https://sites.northwestern.edu/mwds21/)

Northwestern University, Nov 12-14

Titles and abstracts

Paul Apisa

Title: Billiards in Right Triangles and Dynamics on Moduli Space!

Abstract: On a rational right triangle, i.e. one whose angles are all rational multiples of π, how many (bands of) periodic billiard trajectories of length at most L are there? Amazingly, this question is related to dynamics on the moduli space of Riemann surfaces.

Each rational polygon P may be unfolded to a closed surface tiled by copies of P. I will begin by describing how $\mathrm{GL}(2,\mathbb{R})$ acts on the collection of such flat surfaces and (by work of Eskin and Masur) how the $\mathrm{GL}(2,\mathbb{R})$ orbit closure of the unfolding of P controls many dynamical properties of billiard flow on P. I will then explain how to compute the orbit closure of the unfolding of every rational right triangle and describe the consequences it has for billiards.

The proof will build upon new techniques for inductively studying orbit closures introduced by Wright and myself. Other key ingredients in the proof include variational formulas in Teichmuller theory, the work of Eskin and Mirzakhani on orbit closures, and the work of Eskin, Kontsevich, and Zorich on sums of Lyapunov exponents.

Jeffrey Diller

Title: A transcendental dynamical degree

Abstract: The dynamics of a rational map on a compact projective manifold are governed by its so-called dynamical degrees. These numerical invariants are often just easily computed integers, but in general they can be quite hard to evaluate. In this talk, I will survey the definition and significance of dynamical degrees and give some important examples, concluding with one in which the first of the dynamical degrees turns out (surprisingly) to be a transcendental number. This is joint work with Jason Bell, Mattias Jonsson and Holly Krieger.

Yan Mary He

Title: A quantitative equidistribution of angles of multipliers of hyperbolic rational maps

Abstract: In this talk, we will consider the angular component of multipliers of repelling cycles of a hyperbolic rational map in one complex variable. Oh-Winter have shown that these angles of multipliers uniformly distribute in the circle $(-\pi,\pi]$. Motivated by the sector problem in number theory, we show that for a fixed $K \gg 1$, almost all intervals of length $2\pi/K \in (-\pi,\pi]$ contains a multiplier angle with the property that the norm of the multiplier is bounded above by a polynomial in K. This is joint work with Hongming Nie.

Thomas Hille

Title: Bounds for the Least Solution of Quadratic Inequalities.
Abstract: Let Q be a non-degenerate indefinite quadratic form in d variables. In the mid 80's, Margulis proved the Oppenheim conjecture, which states that if $d \geq 3$ and Q is not proportional to a rational form then Q takes values arbitrarily close to zero at integral points. In this talk we will discuss the problem of obtaining bounds for the least integral solution of the Diophantine inequality $|Q[x]| < \epsilon$ for any positive ϵ if $d \geq 5$. We will show

how to obtain explicit bounds that are polynomial in ϵ^{-1}, with exponents depending only on the signature of Q or if applicable, the Diophantine properties of Q. This talk is based on joint work with P. Buterus, F. Götze and G. Margulis.

Steve Hurder

Title: Arboreal Dynamics

Abstract: An arboreal action is an action of a countable group G acting on a bounded-valence tree, preserving a basepoint, or root vertex. The action preserves the distance from the root, and so the levels of the vertices in the tree. The action is minimal if the action is transitive on each level. We assume that all actions are minimal. An arboreal action induces an action on the boundary ends of the tree, and this action is a minimal equicontinuous action on the Cantor set of ends.

Study of the dynamical properties of arboreal actions has applications to:

✻ Structure of Absolute Galois Groups

✻ Classification of Generalized Solenoids

✻ Renormalization and Abstract Commensurators

This talk will survey some recent results and applications of arboreal dynamics in joint works with Olga Lukina and Wouter van Limbeek.

Anh Le

Title: Interpolation sets for nilsequences

Abstract: Interpolation sets are classical objects in harmonic analysis which have a natural generalization to ergodic theory regarding nilsequences. A set E of natural numbers is an interpolation set for nilsequences if every bounded function on E can be extended to a nilsequence on $\mathbb{N}$. By a result of Strzelecki, lacunary sets are interpolation sets for nilsequences. In this talk, I show that interpolation sets for nilsequences are stable under union with finite sets and no sublacunary sets are interpolation sets for nilsequences.

Davi Obata

Title: Uniqueness of the measure of maximal entropy for the standard map

Abstract: The standard family (or Taylor-Chirikov standard family) is a famous example of a family of dynamical systems having a “simple expression” but with complicated dynamics. A famous conjecture by Sinai states that for large parameters the standard map has positive entropy for the Lebesgue measure. In this talk, I will discuss the proof of the

uniqueness of the measure of maximal entropy (m.m.e.) of the standard map for sufficiently large parameters. If time permits, I will also explain some properties for this mme, such as equidistribution of “sufficiently hyperbolic” periodic points and estimates on the dimension of this measure.

Wenyu Pan

Title: Exponential mixing of flows for geometrically finite hyperbolic manifolds with cusps

Abstract: Let $\mathbb{H}^n$ be the hyperbolic n-space and Γ be a geometrically finite discrete subgroup in $Isom_+(\mathbb{H}^n)$ with parabolic elements. We investigate whether the geodesic flow (resp. the frame flow) over the unit tangent bundle $T^1(\Gamma\backslash\mathbb{H}^n)$ (resp. the frame bundle $F(\Gamma\backslash\mathbb{H}^n)$) mixes exponentially. This result has many applications, including spectral theory, orbit counting, equidistribution, prime geodesic theorems, etc.

In the joint work with Jialun LI, we show that the geodesic flow mixes exponentially. I will describe some ingredients in the proof. If there is time, I will also discuss the difficulty of obtaining exponential mixing of the frame flow.

Ralf Spatzier

Title: Hyperbolic Dynamical Systems, Higher Rank and Classification

Abstract: Dynamical systems with extra symmetry turn out to be surprisingly rigid. For instance generically a diffeomorphism cannot have infinite index in its centralizer in the diffeomorphism group, as asserted by Smale and proved by Bonatti, Crovisier and Wilkinson. For hyperbolic systems one can hope to go one step further, and conjecture that such are always smoothly conjugate to an action on a homogeneous space (Katok-Spatzier).

For a special class of hyperbolic actions, the so-called Cartan actions, this was proved recently (Vinhage-S). Most importantly, we introduce a novel way of providing a homogeneous structure to a system coming from actions of free products.
As we will explain, this particular conjecture was motivated in part by the Zimmer program on actions of higher rank semisimple Lie groups and their lattices. And indeed (Butler-Damjanovic-S-Vinhage-Xu) proved a classification result for (totally) Anosov volume preserving actions of such groups, using related tools.

These ideas are closely related to superrigidity, rank rigidity in Riemannian geometry. This will be the setting for the talk, and also the promise and outlook for future work.