Midwest Dynamical Systems Meeting: Oct 30th-Nov 1st 2015

Department of Mathematics, The Ohio State University

Friday

Friday talks will take place in the **auditorium of the MBI** (Mathematical Bioscience Institute), 3rd floor of Jennings Hall, 1735 Neil Ave. Registration and Drinks Reception will take place in the Math Tower 7th floor lounge, 231 W. 18th Ave. It is a 10-15 minute walk between these two locations.

- 2:00 - 2:50pm Registration (7th Floor Lounge, Math Tower)
- 3:00 - 3:50pm **Conference Plenary Talk: Yakov Pesin (Penn State)**
  "The essential coexistence phenomenon in dynamics"
- 3:50 - 4:20pm Break
- 4:20 - 5:10pm **Jon Chaika (University of Utah)**
  "There exists a mixing C^∞ area preserving flow on a surface with all fixed points non-degenerate"
- 5:20 - 7:00pm Drinks Reception (7th Floor Lounge, Math Tower)

Saturday

All Saturday talks will take place in **EA0170** in the 'Math Annex Building', 209 W. 18th Ave., which is connected to the Math Tower. The poster session will take place in the 7th floor lounge of the Math Tower. The conference dinner will take place at the Drake Events Center.

- 9:00-9:50am **Jana Rodriguez Hertz (IMERL, Uruguay)**
  "Center-unstable foliations do not have compact leaves"
- 10:00-10:30am Refreshments
- 10:40-11:30am **Anush Tserunyan (University of Illinois)**
  "Differentiation of subsets of semigroups, a Ramsey theorem and a van der Corput lemma"

Session in honor of the 70th birthday of Carl Simon:

- 11:40-12:30pm **Don Saari (UC-Irvine)**
  "Dynamics and the dark matter mystery"
- 12:30-2:00pm Lunch
- 2:00-2:50pm **Jeff Xia (Northwestern University)**
  "Conley-Zehnder index and some applications in Hamiltonian dynamics"
- 3:00-4:20pm **Poster Session with Refreshments (7th Floor Lounge, Math Tower)**
- 4:30-5:20pm **Charles Pugh (UC-Berkeley)**
  "Unbound Hyperbolicity"
- 5:45-6:30pm Drinks (Drake Events Center, River Den Room)
- 6:30- 9:00pm **Conference Dinner (Drake Events Center, River Den Room)**

Sunday

All Sunday talks will take place in **EA0170**.

- 9:00-9:50am **Keith Burns (Northwestern University)**
  "Unique equilibrium states for geodesic flows in nonpositive curvature"
- 10:00-10:30am Refreshments
- 10:40-11:30am **Zhiren Wang (Penn State)**
  "Mobius disjointness for analytic skew products"
- 11:40-12:30pm **Jim Yorke (University of Maryland)**
  "Chaos and Quasiperiodicity"

Talk Titles and Abstracts (Chronological Order)

Yakov Pesin (Penn State)
The essential coexistence phenomenon in dynamics.

I will discuss two different types of essential coexistence of regular (zero Lyapunov exponents and hence, zero entropy) dynamics and chaotic (non-zero Lyapunov exponents) dynamics in the setting of smooth dynamical systems, both with discrete and continuous time. I will review some recent results in this direction, discuss some open problems and describe some new examples of coexistence. In particular, I will outline a construction of a volume preserving topologically transitive diffeomorphism of a compact smooth Riemannian manifold which is ergodic (indeed is Bernoulli) on an open and dense subset of not full measure and which has zero Lyapunov exponent on the complement of this set. I will also discuss a continuous-time version of this example. These constructions demonstrate a "complete" KAM-type picture in the volume preserving category (in both discrete and continuous-time).

Jon Chaika (University of Utah)
There exists a mixing C^∞ area preserving flow on a surface with all fixed points non-degenerate.

We construct a C^∞ area preserving flow on a surface with all fixed points non-degenerate which is mixing with respect to the volume on the surface. Kocergin showed that this impossible for the torus. C. Ulcigrai showed this was exceptional for any genus. This is joint work with A. Wright.

Jana Rodriguez Hertz (IMERL, Uruguay)
Center-unstable foliations do not have compact leaves.

For a partially hyperbolic diffeomorphism on a 3-manifold, we show that any invariant foliation tangent to the center-unstable bundle has no compact leaves.

Anush Tserunyan (University of Illinois)
Differentiation of subsets of semigroups, a Ramsey theorem and a van der Corput lemma

What often lies at the heart of multiple recurrence results is that for measure-preserving actions of semigroups, mixing along a suitable filter on the semigroup amplifies itself to multiple mixing along the same filter. This amplification is usually proved using a so-called van der Corput difference lemma. Instances of this lemma for specific filters have been proven before by Furstenberg, Bergelson-McCutcheon, and others, with a somewhat different proof in each case. We define a notion of differentiation for subsets of semigroups and isolate a class of filters that respect this notion. The filters in this class (call them ∂-filters) include all those, for which the van der Corput lemma was known, and our main result is a van der Corput lemma for ∂-filters, which thus generalizes its previous instances. This is done via proving a Ramsey theorem for graphs on the semigroup.

Don Saari (UC-Irvine)
Dynamics and the dark matter mystery

One of the more compelling contemporary mysteries in science is that expected, but still undiscovered, dark matter that supposedly makes up most of the mass of a galaxy. While this issue appears to be one strictly for astrophysicists, in fact it is an issue for mathematicians and dynamical systems. It will be shown why this is so and why, if huge amounts of dark matter are eventually discovered, how it will create a problem for Newtonian mechanics.

Jeff Xia (Northwestern University)
Conley-Zehnder index and some applications in Hamiltonian dynamics

The Conley-Zehnder index is a Maslov type of index introduced to study the periodic solutions of the Hamiltonian systems. These indices help to connect the periodic orbits to the topology of underlying manifold. Here we use the index to study the stability and the bifurcations of the periodic solutions. We give a simple and interesting condition for the instability of some periodic orbits, we also give the generic bifurcation scenarios when the Conley-Zehnder index of a periodic orbit changes in a one-parameter family. This is a joint work with Yanxia Deng.

Charles Pugh (UC-Berkeley)
Unbound Hyperbolicity

I'll discuss some ideas about invariant manifolds for an endomorphism of a Banach space where the linear part is unbounded.

Keith Burns (Northwestern University)
Unique equilibrium states for geodesic flows in nonpositive curvature

The geodesic flow for a compact Riemannian surface with negative curvature has a unique equilibrium state for every Hölder continuous potential function. This is no longer true if the curvature of the surface is only nonpositive, but there is a still large class of potentials with unique equilibrium states. These include multiples of the geometric potential. This is joint work with Vaughn Climenhaga, Todd Fisher and Dan Thompson.

Zhiren Wang (Penn State)
Mobius disjointness for analytic skew products

Sarnak's Mobius disjointness conjecture states that the Mobius function is disjoint to every topological dynamical system of zero entropy. In this talk, we will explain why this is true for every analytic skew product map on the two torus over a rotation of the circle of arbitrary rotation number.

Jim Yorke (University of Maryland)
Chaos and Quasiperiodicity

Chaos and quasiperiodicity are the two kinds of recurrent nonlinear dynamics that one is likely to have in generic dynamical systems. As part of our numerical studies we found a rather surprising (rigorous) result. The Birkhoff Ergodic Theorem concludes that time averages, i.e., Birkhoff averages, $\Sigma_{n=0}^{N-1} f(x_n)/N$ of a function f along a length N ergodic trajectory (x_n) of a function T converge to the space average $\int f d\mu$, where μ is the unique invariant probability measure. Convergence of the time average to the space average is slow, like $1/N$ or slower. We introduce a modified average of $f(x_n)$ by giving very small weights to the "end" terms when n is near 0 or $N-1$. When (x_n) is a trajectory on a quasiperiodic torus and f and T are C^∞, we show that our weighted Birkhoff averages converge "super" fast to $\int f d\mu$ with respect to the number of iterates N, *i.e.* with error $|\Sigma_{n=0}^{N-1} f(x_n)/N - \int f d\mu|$ decaying faster than N^{-m} as $N \to \infty$ for every $m > 0$. We show that our weighted Birkhoff average is a powerful computational tool for computing rotation numbers and conjugacies. This is joint work with work with Suddhasattwa Das, Yoshitaka Saiki and Evelyn Sander.

Poster Session Titles and Abstracts (Alphabetical Order)

Sarah Bray (Tufts)

Ergodic geometry for nonstrictly convex Hilbert geometries

In his work on convex, open, bounded Ω in projective space with compact quotients by discrete groups of projective transformations, Benoist proves that in many ways the strictly convex Ω generalize hyperbolic space when endowed with the Hilbert geometry. Dynamically, he and Crampon have independently shown that the geodesic flow on the unit tangent bundle of the quotient manifold exhibits classical uniformly hyperbolic behavior (it is an Anosov flow) and hence the measure of maximal entropy is unique.

For a nonstrictly convex Hilbert geometry, new techniques are needed. Benoist proved under mild hypotheses that for nonstrictly convex Hilbert geometries in dimension three, compact quotients have embedded flats as boundary tori or Klein-bottles to hyperbolic components. We explore how techniques from rank one manifolds and nonuniformly hyperbolic dynamics need to be and can be adapted for these examples, with emphasis on the power of geometry for proving hyperbolic dynamical phenomena. Main results include positive entropy, existence and uniqueness of conformal measures at infinity, divergence of the group, and ergodicity of Bowen-Margulis measure for the geodesic flow.

Marta Canadell (ICERM, Brown)

Computation of normally hyperbolic invariant manifolds

In this poster we explain a method for the computation of normally hyperbolic invariant manifolds (NHIM) in discrete dynamical systems. The method is based on finding a parameterization for the manifold by formulating a functional equation. We solve the invariance equation using a Newton-like method taking advantage of the dynamics and the geometry of the invariant manifold and its invariant bundles. Particularly, we present two different methods to compute normally hyperbolic invariant tori (NHIT). The first method is a KAM-like theorem in a-posteriori format for the existence of quasi-periodic invariant tori in smooth families of real-analytic dynamical systems, which provides us an efficient algorithm for computing NHIT, by adjusting parameters of the family. The second method allows us to compute a NHIT and its internal dynamics, which is a-priori unknown. We implement both methods to continue invariant tori with respect to parameters, and to explore different mechanisms of breakdown. This is a joint work with Alex Haro.

Dong Chen

Positive metric entropy arises near a flat metric on 5-dimensional torus

In 2014, D. Burago and S. Ivanov constructed a Finsler metric on S^n $(n > 3)$ that is C^∞ close to the standard metric but with positive metric entropy. In this poster we show that similar case happens on torus. Namely, for arbitrary n, we can perturb the flat metric on T^5 in the class of C^n reversible Finsler metric so that the resulting metric has positive metric entropy.

Xue Gong (Ohio University)
Heteroclinic binding networks—a model in sequential memory

Temporal order memories are critical for everyday animal and human functioning. Some evidence has shown that the binding or association of various features of an event together and the maintaining of multimodality events in sequential order is the key component of any sequential memories. We studied the mathematical aspect of this using a high-dimensional model in the generalized form of the Lotka-Volterra equations. In the phase space of the model there exists a multi-dimensional binding heteroclinic network consisting of a heteroclinic chain of heteroclinic cycles. We proved the robustness of the binding sequential dynamics, i.e., the feasibility phenomenon for coupled heteroclinic networks: for each collection of successive heteroclinic trajectories inside the unified networks, there is an open set of initial points such that the trajectory going through each of them follows the prescribed collection staying in a small neighborhood of it. We show also that the symbolic complexity function of the system restricted to this neighborhood is a polynomial. Therefore, the symbolic system is not chaotic.

Charles Jaffé (West Virginia University)
Relative Equilibria in the Semiclassical Coulomb problem

I discuss the nature of the Semiclassical Coulomb problem and then turn my attention the relative equilibria in these systems. I review what is known concerning relative equilibria in atoms and molecules. I then turn my attention to the role that relative equilibria play in chemical reactions.

John Lesieutre (UIC)
Which Three-Dimensional Algebraic Varieties Admit Positive Entropy Automorphisms?

There are many examples of smooth, algebraic surfaces admitting (biholomorphic) automorphisms of positive entropy. However, there are few known examples of such automorphisms on higher-dimensional varieties. I will present some results which partly explain this scarcity.

Bingbing Liang (Buffalo)
Sofic mean topological dimension of algebraic actions

For any discrete group Γ, there is a one-to-one correspondence between $\mathbb{Z}\Gamma$-modules M and the topological dynamical systems $\Gamma \curvearrowright \widehat{M}$ by continuous autumorphisms (so-called *algebraic actions*). The sofic mean topological dimension is a numerical invariant in dynamics and is related to entropy. The von Neumann-Lück rank is a L^2-invariant and is related to L^2-Betti number.

Assume Γ is a sofic group, M is a finitely-presented $\mathbb{Z}\Gamma$-module such that $\widehat{M}$ is an absolute neighborhood retract (ANR), we show that the sofic mean topological dimension of $\widehat{M}$ equals to the von Neumann-Lück rank of M. Although the condition of being an ANR does not hold in general, we show it has some connection to the expansiveness of dynamical systems.

Olga Lukina (UIC)
The discriminant invariant of Cantor group actions.

Abstract: In this paper, we consider minimal equicontinuous actions of non-abelian groups on Cantor sets. Such an action Φ may be classified as *regular*, *weakly regular* or *irregular* by the properties of its automorphism group, which reflects the degree of non-homogeneity of the action. We introduce an invariant, called the *discriminant function* $\mathcal{D}(\Phi)$, which assigns to each point $x \in X$ a profinite group $\mathcal{D}_x$. The cardinality of $\mathcal{D}_x$ does not depend on the point and is related to the degree of non-homogeneity. We characterize the actions for which the discriminant group is finite, and give new examples of minimal actions for which $\mathcal{D}_x$ is not trivial. Joint work with Jessica Dyer and Steve Hurder.

Lien-Yung Kao (Notre Dame)
Entropy, critical exponent and immersed surfaces in hyperbolic 3-manifolds

We consider a π_1–injective immersion $f : \Sigma \to M$ from a compact surface Σ to a hyperbolic 3–manifold M. Let Γ denote the copy of $\pi_1\Sigma$ in $\mathrm{Isom}(\mathbb{H})^3$ induced by the immersion f, and we endow Σ with the induced metric. Using the thermodynamic formalism, when Σ is negatively curved and Γ is convex cocompact, we prove an inequality relating the topological entropy $h(\Sigma)$ of the geodesic flow on T^1M and the critical exponent δ_Γ of Γ: $C_1(\Sigma, M)\cdot\delta_\Gamma \leq h(\Sigma) \leq C_2(\Sigma, M)\cdot\delta_\Gamma$, where $C_1(\Sigma, M)$ and $C_2(\Sigma, M)$ are two geometric constants. Herein, we investigate geometry meanings of these two constants in detail. Furthermore, we study the rigidity phenomenon arising from this inequality. Lastly, we apply our results to immersed minimal surfaces in hyperbolic 3–manifolds, and these discussions lead us to a similar study as A. Sanders' work on the moduli space of Σ introduced by C. Taubes.

Mehrzad Monzavi (Univeristy of Texas at Arlington)
On Fixed Point Property of Nilpotent and Solvable Lie Group Actions on Nonpositively Curved Compact Manifolds

We will prove the following theorems. The first theorem posits the existence of a fixed point for the actions of nilpotent lie groups on nonpositively curved compact manifolds. The second theorem states that actions of solvable Lie groups on nonpositively curved compact manifolds have either a fixed point or a 2-periodic point.

Cara Mullen (UIC)
p-adic Hubbard Trees

In complex dynamics, Hubbard trees offer a combinatorial description of the dynamics of post-critically finite (PCF) polynomials. What are the analogous objects in a non-Archimedean setting: what is a p-adic Hubbard tree? We explore this question by studying the critical orbit trees associated to quadratic maps $f_c(z) = z^2 + c$, with $c \in \mathbb{Z}_p$ (for $p > 2$).

Rob Niemeyer (University of Maine)
Nontrivial paths and periodic orbits of the T-fractal billiard.

In this poster presentation, we: 1) determine some periodic orbits and nontrivial paths of the T-fractal billiard table, 2) construct a forward singular orbit that behaves like particular periodic orbits, and 3) present future directions of research.

Anca Radulescu (SUNY New Paltz)

Real and complex behavior for networks of coupled logistic maps

Many natural systems are organized as networks, in which the nodes interact in a time-dependent fashion. The object of our study is to relate connectivity to the temporal behavior of a network in which the nodes are real or complex logistic maps, coupled according to a connectivity scheme that obeys certain constrains, but also incorporates random aspects. We discuss the possibility of extending Fatou-Julia theory to such ensembles of maps coupled as nodes in a network.

We investigate in particular the relationship between the system architecture and dynamics. We illustrate how the system's behavior (measured via topological properties of the Julia set) changes when perturbing the underlying adjacency graph. We differentiate between the effects on dynamics of the following operations which directly modulate network connectivity: (1) increasing/decreasing edge weights, (2) increasing/decreasing edge density, (3) altering edge configuration by adding, deleting or moving edges.

Yuki Takahasi (UC-Irvine)

Products of two Cantor sets and application to the Labyrinth model

We consider products of two Cantor sets, and obtain the optimal estimates in terms of thickness that guarantee that their product is an interval. This problem is motivated by the fact that the spectrum of the Labyrinth model, a two dimensional quasicrystal model, is given by the product of two Cantor sets. We also discuss the connection between this question and "intersection of two Cantor sets" problem, which was considered in several papers before.