2011 Midwest Dynamical Systems Seminar
IUPUI, October 14-16

All talks are in IT 252.

Friday 10/14:

2:30pm–3:00pm	**Coffee Break**
3:00pm–4:00pm	**John Hubbard** Parabolic blowups for polynomials and Kleinian groups.
4:00pm–4:30pm	**Coffee Break**
4:30pm–5:30pm	**Tanya Firsova** Critical Locus for Complex Hénon Maps.

Saturday 10/15:

9:00am–10:00am	**Stephen Chamblee** Twisted Tent Maps
10:00am–10:30am	**Coffee Break**
10:30am–11:30am	**Piotr Oprocha** On syndetically proximal relation and scrambled sets
11:40am–12:40pm	**Andrey Gogolev** Partially hyperbolic diffeomorphisms with compact center foliation
12:40pm–2:40pm	**LUNCH break**
2:40pm–3:40pm	**Marco Martens** Renormalization in low dimensional dynamics
3:50pm–4:50pm	**Yulij Ilyashenko** Bony and thick attractors
4:50pm–5:20pm	**Coffee Break**
5:20pm–6:20pm	**Eva Uhre** Limits of polynomial-like quadratic rational maps and stars in attracting basins
7:30pm–??	**PARTY at Will Geller's house.** 1739 Brewster Rd, Indianapolis.

Sunday 10/16:

9:00am–10:00am	**Lluís Alsedà** An example of a strongly invariant pinched core strip for quasiperiodically forced skew products on the annulus
10:00am–10:30am	**Coffee Break**
10:30am–11:30am	**Ana Rodrigues** Uniform Hyperbolicity for Double Standard Maps
11:45am–12:45pm	**Enrique Pujals** Some simple questions and results related to the C^r stability

Abstracts:

Lluís Alsedà: An example of a strongly invariant pinched core strip for quasiperiodically forced skew products on the annulus. (In collaboration with F. Mañosas and L. Morales)

Abstract: We will construct an example of a quasiperiodically forced skew product on the annulus which has a 2-periodic orbit of curves and a strongly invariant pinched core strip (in the sense of [FJJK]) that does not contain any arc of a curve. Moreover, our example is monotone (decreasing) on the fibres and the pinched set has Lebesgue measure one. However, it is not a continuous curve.

[FJJK] R. Fabbri, T. Jäger, R. Johnson and G. Keller, *A Sharkovskii-type theorem for minimally forced interval maps*, Topological Methods in Nonlinear Analysis, Journal of the Juliusz Shauder Center, **26** (2005), 163–188.

Stephen Chamblee: Twisted Tent Maps

Abstract: A twisted tent map is a complex generalization of a real tent map. The action of this map can be visualized as the complex scaling of the plane followed by folding the plane once. Most of the time, scaling by a complex number will twist the plane, hence the name. The folding both breaks analyticity (and even smoothness) and leads to interesting dynamics ranging from easily understood and highly geometric behavior to chaotic behavior and fractals.

Tanya Firsova: Critical Locus for Complex Hénon Maps.

Abstract: Complex Hénon maps are maps of the form $(x,y)\mapsto(x^2+c-ay,x)$. In one-dimensional dynamics, critical points behavior to large extent capture the global dynamics of the map. Hénon maps are biholomorphisms of $\mathbb{C}^2$. They do not have critical points in the classical sense. Rather, the critical locus is defined in the following way: Within the forward and backward escaping loci $U_a^{\pm}$ there are dynamically defined foliations $\mathcal{F}_a^{\pm}$. The level sets of the Green function G_a^+ on U_a^+ are foliated by holomorphic curves, giving $\mathcal{F}_a^+$. Two points belong to the same leaf, if their orbits converge, as they escape to infinity. The foliation $\mathcal{F}_a^-$ is defined analogously. The critical locus is the set of tangencies between these two foliations. We give a description of the critical locus for Hénon maps that are small perturbations of the quadratic polynomials with disconnected Julia set. We justify the picture conjectured by John Hubbard.

Andrey Gogolev: Partially hyperbolic diffeomorphisms with compact center foliation

Abstract: A diffeomorphism of a smooth manifold M is called partially hyperbolic if the tangent bundle of M splits into a direct sum of an expanding subbundle, a contracting subbundle and a center subbundle with an intermediate growth rate. By now statistical properties of partially hyperbolic systems are fairly well understood. We will discuss classification of partially hyperbolic systems. In general, this is a hopeless problem. We will consider partially hyperbolic diffeomorphisms whose center bundles integrate to foliations with compact leaves and attempt to reduce classification of such systems to classification of Anosov diffeomorphisms. Our results provide progress on a conjecture of Pugh.

John Hubbard: Parabolic blowups for polynomials and Kleinian groups.

Abstract: Polynomials of a given degree have an obvious topology, as do finitely generated Kleinian groups. The Julia set of a polynomial and the limit set of a Kleinian group do not depend continuously on the polynomial or the Kleinian group.

So an obvious question is: what is the closure of the set of Julia sets, or the set of limit sets, in the Hausdorff topology?

Presumably this question is too hard in general, but more specically, we might consider the closure of the set of filled-in Julia sets K_c for polynomials $p_c : z \mapsto z^2 + c$, or of limit sets for groups representing once punctured tori, appropriately normalized.

Here the problem is approachable, though the answer is still pretty elaborate: the closure is in both cases a projective limit of *parabolic blowups*. The parabolic blowup is a process somewhat analogous to the standard blowups of algebraic geometry: you remove a point and replace it by something else. But the parabolic blowup is definitely a transcendental operation, and the spaces created are pretty bizarre. In my lecture I will try to explain this construction.

Yulij Ilyashenko: Bony and thick attractors

Abstract: Understanding of the structure of attractors of generic dynamical systems is one of the major goals of the theory of these systems. A vast general program suggested by Palis presents numerous conjectures about this structure. Various particular cases of these conjectures are proved in numerous papers that we do not quote here. Main part of these investigations is related to diffeomorphisms of closed manifolds.

Our investigation is in a sense parallel to this direction of research. In the first part of the talk, attractors of manifolds with boundary onto themselves are studied. At present, locally generic properties of attractors of such maps are established, that are not yet observed (and plausibly do not hold) for the case of closed manifolds. For instance, an open set of diffeomorphisms of manifolds with boundary onto themselves may have attractors with intermingled basins [3], [1], [2]. The strongest result of this kind is obtained by Kleptsyn and Saltykov in a paper accepted to the Proceedings of the MMS.

Another property of this kind is having thick attractors. It is a general belief that attractors of typical smooth dynamical systems (diffeomorphisms and flows) on closed manifolds, either coincide with the whole phase space, or have Lebesgue measure zero. In this talk we show that this is not the fact for diffeomorphisms of manifolds with boundary onto themselves. Namely, in the space of diffeomorphisms of a product $\mathbb{T}^2 \times I, I = [0, 1]$, there exists an open set such that any map from a complement of this set to a countable number of hypersurfaces, has a thick attractor: a transitive attractor that has positive Lebesgue measure together with its complement.

The problem, whether or not thick attractors exist for locally generic diffeomorphisms of a closed manifold, remains widely open. In the second part we study so called bony attractors. These are attractors of skew products over a Bernoulli shift with the following unexpected property: the map has an invariant manifold, and the intersection of the attractor with that manifold, called a bone, is much larger than the attractor of the restriction of the map to the invariant manifold. Bony attractors with one-dimensional bones were discovered in [4]. We construct bones of arbitrary dimension.

It is expected that bony attractors are in a sense locally generic in the space of diffeomorphisms of a closed manifold.

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[1] C. Bonatti, L. Diaz, M. Viana, *Dynamics beyond uniform hyperbolicity*, Springer, Berlin, 2004

[2] Yu. Ilyashenko, V. Kleptsyn, P. Saltykov *Openness of the set of boundary preserving maps of an annulus with intermingled attracting basins*, JFPTA, 3, (2008), 449-463.

[3] I. Kan *Open sets of diffeomorphisms having two attractors, each with everywhere dense basin*, Bull. Amer. Math. Soc., 31 (1994) 68-74.

[4] Kudryashov Yu. G. *Bony attractors*, Func. Anal. Appl. 44 (2010), no. 3, 73-76.

Marco Martens: Renormalization in low dimensional dynamics

Abstract: Renormalization in dynamics explains the relation between various aspects: combinatorics, universality of geometry in parameter space, rigidity of the attractor and ergodic theoretical properties. It has played a crucial role in unimodal dynamics and starts to shed some light on Lorenz dynamics, dissipative and conservative Hénon dynamics. However, with some surprises.

Piotr Oprocha: On syndetically proximal relation and scrambled sets
(joint work with T.K. Subrahmonian Moothathu)

Abstract: Let $f\colon X \to X$ be a continuous map acting on a compact metric space (X, d). The *asymptotic*, *proximal*, and *syndetically proximal* relations for f, are defined respectively as

$$\begin{aligned} \mathrm{Asy}(f) &= \{(x,y) \in X^2 : \lim_{n\to\infty} d(f^n(x), f^n(y)) = 0\}, \\ \mathrm{Prox}(f) &= \{(x,y) \in X^2 : \liminf_{n\to\infty} d(f^n(x), f^n(y)) = 0\}, \\ \mathrm{SyProx}(f) &= \Big\{(x,y) \in X^2 : \\ &\qquad \{n \in \mathbb{N} : d(f^n(x), f^n(y)) < \varepsilon\} \text{ is syndetic for all } \varepsilon > 0\Big\}. \end{aligned}$$

Recall that a set S is *scrambled* (resp. *syndetically scrambled*) if $S \times S \setminus \Delta \subset \mathrm{Prox}(f) \setminus \mathrm{Asy}(f)$ (resp. $S \times S \setminus \Delta \subset \mathrm{SyProx}(f) \setminus \mathrm{Asy}(f)$), where Δ is the diagonal in $X \times X$.

One of nice properties of $\mathrm{SyProx}(f)$ is that it is an equivalence relation. From the other point of view, syndetically proximal relation most often is a first category subset of $X \times X$, so it may be hard to deal with it. For example Kuratowski-Mycielski Theorem, which is one of the most effective tools in construction of scrambled sets cannot be applied.

The aim of this talk is to survey known results on syndetically proximal relation and present some tools which can be used for construction of syndetically scrambled sets. We will also examine more concrete classes of dynamical systems, like maps on the unit interval or subshifts.

Enrique Pujals: Some simple questions and results related to the C^r stability conjecture

Abstract: We will expose some questions related to the C^r structural stability conjecture for surface diffeomorphisms, and we will try to explain some partial results.

Ana Rodrigues: Uniform Hyperbolicity for Double Standard Maps

Abstract: In this paper we continue the investigation of the family of *double standard maps* (see Misiurewicz and Rodrigues, Double Standard Maps. *Comm. Math. Phys.* **273**, 37–65 (2007).) of the circle onto itself, given by $f_{a,b}(x) = 2x + a + (b/\pi)\sin(2\pi x) (\mathrm{mod}\ 1)$, where $1/2 < b \leq 1$ and a is a real parameter $0 \leq a < 1$. We will prove that for $b < 1$ but close to $b = 1$ this family possesses an open set in the parameter space so that $f_{a,b}$ is uniformly hyperbolic. In the case $b = 1$ we will prove simultaneously a version of Jakobson's Theorem. This is joint work with M. Benedicks.

Eva Uhre: Limits of polynomial-like quadratic rational maps and stars in attracting basins

Abstract: We discuss the limiting behavior of sequences of polynomial-like quadratic rational maps. One tool for this is the construction of a so called *star* in the immediate basin of an attracting fixed point. The concept of a star in the immediate basin of an attracting fixed point was developed by C. L. Petersen as a tool for giving bounds on multipliers of periodic orbits, with given rotation numbers, for quadratic rational maps. Opposite to the case of a super attracting fixed point, there is no canonical choice of rays, or a foliation, in the immediate basin of an attracting fixed point. A star is a construction that allows to define a system of rays, that can be related to a system of rays in parabolic basins.